Name————————————, ID ——————

Exam I: MTH 221, Fall 2015

Ayman Badawi

QUESTION 1. (i) Given A is a 10×10 matrix such that $det(A) = 0$. Let B be the second column of A and consider the system of linear equations $AX = B$. Then

a. The system has no solutions (inconsistent).
b. $\{(0, 1, 0, 0)\}$ is the solution set to the system.
c. The system has infinitely many solutions.
d. None of the above is correct.

(ii) Let $A = \begin{bmatrix} 0 & a & b \\ 0 & -2 & c \\ 0 & 0 & 1 \end{bmatrix}$. Then $det(A + 3I_3) =$ is

(a) 9 (b) 27 (c) 12 (d) 3

(iii) Let $A = \begin{bmatrix} 2 & 4 & -1 & 2 \\ -3 & 7 & 11 & 23 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 22 \\ -4 & 3 & 15.2 \\ 9 & 0 & 77.5 \\ -7 & 0 & 88 \end{bmatrix}$. Let $D = AB$. Then the second column of D is

(a) $\begin{bmatrix} 12 \\ 21 \end{bmatrix}$ (b) $\begin{bmatrix} 21 \\ 9 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 21 \end{bmatrix}$ (d) Something else

(iv) Given A is a 4×4 matrix and $A \xrightarrow{2R_1 + R_3 \to R_3} A_1 \xrightarrow{3R_4} \begin{bmatrix} 1 & 3 & 4 & -1 \\ 0 & 3 & 1 & 8 \\ 0 & -3 & 2 & 1 \\ -1 & -3 & -4 & 4 \end{bmatrix}$. Then $det(A^T)$

(a) 9 (b) 27 (c) $\frac{1}{9}$ (d) 51

(v) Given A is a 2×3 matrix and $A \xrightarrow{-2R_1 + R_2 \to R_2} B \xrightarrow{4R_2} D = \begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \end{bmatrix}$. Let F, W be two elementary matrices such that $FWA = D$. Then

(a) $W = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, (b) $F = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ (c) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) Something else

(vi) Given A is a 3×3 matrix and $A \xrightarrow{-2R_1 + R_2 \to R_2} B \xrightarrow{4R_2} D = \begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 1 & 2 & 62 \end{bmatrix}$. Then

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A =$$

(a) $\begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 1 & 2 & 62 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 0 & -1 & -4 \end{bmatrix}$. (c) D^T (d) Something else

(vii) Given A is nonsingular matrix such that $A^{-1} = \begin{bmatrix} a & -3 & 2 \\ b & 1 & 1 \\ c & 0 & -3 \end{bmatrix}$, for some fixed numbers a, b, c. Consider the system of linear equations $AX = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Then

(a) It is possible that the system has no solution. (b) $\{(1, 3, -6)\}$ is the solution set of the system
(c) $\{(a-6, b+2, c)\}$ is the solution set of the system (d) Not enough information in order to determine the solution set, but I am sure that it must have a unique solution.

(viii) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$. Then A^{-1} =

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ (c) There is no inverse of A (d) Something else

(ix) Given $A = \begin{bmatrix} a & 2 & 0 \\ b & 1 & 4 \\ c & 0 & d \end{bmatrix}$ such that $det(A) = -4$ (i.e, A is invertible). Then the (1, 3)-entry of A^{-1} is

(a) $\frac{c}{-4}$ (b) $\frac{c}{4}$ (c) -2 (d) something else

(x) Let $A = \begin{bmatrix} 2 & 4 & a \\ 0 & 2 & b \\ 0 & -2 & d \end{bmatrix}$ such that $det(A) = 2$. Consider the system $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$. Then the value of x_3

(a) cannot be determined, I need more info. (b)5 (c) 10 (d) 6 (e) Something else

(xi) Let $A = \begin{bmatrix} 2 & a & b \\ -2 & 4 & 7 \\ 4 & 2a & 10 \end{bmatrix}$. The values of a, b where the system $AX = \begin{bmatrix} \sqrt{7} \\ 2015 \\ 36.23 \end{bmatrix}$ has unique solution are :

(a) $a \neq -4$ and $b \neq 5$ (b) $a \neq 0$ and b can be any real number (c) $a = 0$ and $b \neq 0$ or $b = 0$ and $a \neq 0$. (d) Something else

(xii) Given A is a 2×2 matrix such that $A \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} - 2A = I_2$. Then $A =$

(a) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (c) $\frac{1}{3} \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ (d) Something else

(xiii) Given the augmented matrix of a system of linear equations $\begin{bmatrix} 2 & 4 & 2 & 0 & 4 \\ -1 & -2 & -1 & 1 & -1 \\ 3 & 6 & 4 & 0 & 3 \\ -1 & -2 & -1 & 0 & -2 \end{bmatrix}$. The solution set of the system is

(a) $\{(5 - 2x_2,\ x_2,\ -3,\ 1) \mid x_2 \in R\}$ $\{(-1 + x_4,\ 1,\ -3,\ x_4) \mid x_4 \in R\}$ $\{(5,\ 0,\ -3,\ 1)\}$ (d) Something else

(xiv) Given the augmented matrix of a system of linear equations $\begin{bmatrix} 1 & -1 & 3 & 4 \\ -1 & 1 & a & 6 \\ -2 & 2 & b & -8 \end{bmatrix}$. The values of a, b that make the system consistent (i.e, has a solution)

(a) $a \neq -3$ and $b = -6$ (b) a can be any real number, $b \neq -6$ (c) $a = -3$ and $b = -6$ (d) Something else.

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Exam II , MTH 221, Fall 2015

Ayman Badawi

QUESTION 1. (i) Let $D = \{(a + b + 2c,\ 3a - 3b,\ a + 2b + 3c) \mid a, b, c \in R\}$. Then $dim(D) =$

a) 1 b) 2 c)3 d) None

(ii) Let A be a 2×2 matrix such that A is row-equivalent to $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$. Then the eigenvalues of A are :

a) $2, 4$ b)$\frac{1}{2}, \frac{1}{4}$ c) $\frac{1}{2}, 4$ d) None of the previous

(iii) Which of the following matrices with the given properties are (is) INVERTIBLE and diagnolizable:

a)A is 3×3, $C_A(x) = (x-3)^2(x-4)$, $E_3 = span\{(2, 0, 2), (0, 1, 4)\}$, and $E_4 = span\{(0, 0, 9)\}$

b) A is 2×2, $C_A(x) = (x - 4)^2$ and $E_4 = span\{(0, 7)\}$

c) A is 2×2, $C_A(x) = x(x - 2)$, $E_0 = span\{(4, 1)\}$, and $E_2 = span\{(0, 5)\}$

d) a and b (e) b and c (f) a and c

(iv) Let $A = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & -1 & -4 \\ 0 & 0 & -3 & -12 \end{bmatrix}$. Then $N(A) =$

a) $span\{(0, 0, 1, 4), (1, 0, 0, 0)\}$ b) $span\{(0, 2, 0, 0)\}$ c) $span\{(0, 1, 0, 0), (0, 0, -4, 1\}$ d) None of the previous

(v) Let $A = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & -1 & -4 \\ 0 & 0 & -3 & -12 \end{bmatrix}$. Then $col(A)$

a) Span $\{(0, 1, 0), (4, -4, -12)\}$ b) Span $\{(0, 1, 0), (1, 0, 0)\}$ c) span$\{(1, -1, -2)\}$ d) None of the previous

(vi) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -6 \\ 0 & 1 & -5 \end{bmatrix}$. Then the eigenvalues of A are :

a) 0, -5 b) 0, -2, -3 c) 0, -6, -5 d) 1, -5, -6 e)None of the previous

(vii) Let D be a subspace of $R^{2\times 2}$ such that $dim(D) = 2$. Then a possibility for D is

a) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ 0 & 0 \end{bmatrix} \mid a, b \in R\}$ b) $D = \{\begin{bmatrix} a+3 & 4a \\ b & 6b \end{bmatrix} \mid a, b \in R\}$

c) $D = \{\begin{bmatrix} a+2b+c & 3a+6b \\ c & 0 \end{bmatrix} \mid a, b, c \in R\}$ d) $D = \{\begin{bmatrix} a+2b & 2a+4b \\ c & a+b \end{bmatrix} \mid a, b, c \in R\}$

(viii) Let A be a 2×2 matrix such that $A\begin{bmatrix} 1 \\ 9 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ and $det(A) = 15$. Then Trace(A) =

a) 6 b) 8 c) 30 d) 10 e) Need more information.

(ix) Given $D = \{(a, b, c) \in R^3 \mid a + b = 0$ and $a + c = 0$, where $a, b, c \in R\}$ is a subspace of R^3. Then $D =$

a) $span\{(1, 0, -1), (1, -1, 0)\}$ b) $span\{(-6, 6, 6)\}$ c) $span\{(0, 1, -1), (1, -1, 0)\}$ d) R^3 e)None of the previous

(x) One of the following is a basis for P_3

a)$\{1+x^2, -x-x^2, x^2\}$ b)$\{1+x+x^2, -1-x-2x^2, 1+x+5x^2\}$ c) $\{5, x-3x^2, 10+3x-9x^2\}$ d) $\{10, x+3, 16+2x\}$ e) None of the previous is correct

(xi) Given $F=\{f(x)\in P_4 \mid f'(2)=0\}$ is a subspace of P_4. Then a basis for F is

a) $\{x-2, x^2-4, x^3-8\}$. b) $\{x^2-4x,\ x^3-12x\}$ c) $\{x^3+x^2-16x,\ x^3-3x^2,\ x^2-4x\}$ d) None of the previous

(xii) One of the following is true:

a) $\{A\in R^{2\times 2} \mid det(A^T)=0\}$ is a subspace of $R^{2\times 2}$ b) $\{(a,b,c)\in R^3 \mid a,b,c\in R\ \ and\ \ a+b+c-1=0\}$ is a subspace of R^3

c) $\{(a^3, b, a^3) \mid a,b\in R\}$ is a subspace of R^3 d) $\{(a, 3a+b, -b) \mid a,b\geq 0\}$ is a subspace of R^3.

(xiii) Given that $S=\{A\in R^{2\times 2} \mid Trace(A)=0\}$ is a subspace of $R^{2\times 2}$. Then $dim(S)=$

a) 4 b) 3 c) 1 d) 2

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Final exam: MTH 221, Fall 2015

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FINAL EXAM V2

Name: ____________________

ID: ____________________

Section: ____________________

- Write all steps clearly. Otherwise points might be deducted.
- Mobiles are not allowed in this exam.

Question #	Marks	Maximum Marks
Q1		16
Q2		8
MCQ		54
TOTAL		80

$\boxed{Q1}$ Let $A = \begin{bmatrix} 6 & 4 & 2 \\ 4 & 6 & 2 \\ 0 & 0 & 2 \end{bmatrix}$. Is A diagnolizable? If yes, then find a diagonal matrix D and an invertible matrix Q such that $D = Q^{-1}AQ$. **Do not find** Q^{-1}.

$\boxed{Q2}$ Let $V = span\{(1, 0, 2), (-1, 0, 1)\}$. Use Gram-Schmidt process to construct an orthogonal basis for V.

MCQ V.2

- The **Capital Letter** that corresponds to your answer choice should be clearly written in the middle column.
- Only one answer choice per question will be accepted.

Question #	Your answer	Marks
Q1		4
Q2		4
Q3		4
Q4		4
Q5		4
Q6		4
Q7		4
Q8		4
Q9		4
Q10		4
Q11		4
Q12		4
Q13		4
Q14		4
TOTAL		54

Q1 Let $M = \begin{bmatrix} -1 & -2 & 1 & 3 \\ 1 & 0 & 1 & -1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$. Then the COMPLETE reduced form of of M is

A) $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -2 & 2 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{Q2}$ Let M be a $n \times n$- matrix such that $\det(M) \neq 0$. Which of the following statements is **always true**

A) M is diagonalizable

B) M has n distinct eigenvalues

C) 0 is an eigenvalue of M

D) It is possible that 0 is an eigenvalue of A^T

E) All eigenvalues of M are nonzero

Q3 If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is such that $\det A = 4$, then the determinant

$$\begin{vmatrix} a-2d & b-2e & c-2f \\ \frac{1}{2}g & \frac{1}{2}h & \frac{1}{2}i \\ 2d & 2e & 2f \end{vmatrix}$$ is equal to

A) 8

B) 4

C) 2

D) -8

E) -4

$\boxed{Q4}$ Consider the following subsets of $\mathcal{P}_3$:

$$R = \{f(x) \in \mathcal{P}_3 : \ f'(2) = 0\}, \ S = \{f(x) \in \mathcal{P}_3 : \ f(1) \geq 0\}$$

$$\text{and } T = \{f(x) \in \mathcal{P}_3 : \ f(x) + f'(x) = 0\}.$$

Which of these subsets is a subspace of $\mathcal{P}_3$?

A) R, S, and T

B) R and T only

C) T only

D) S only

E) R only

Q5 Recall that a square matrix A is said to be symmetric if $A^t = A$. If A is a square matrix, then

A) AA^t and $A - A^t$ are symmetric

B) $A + A^t$ and $A - A^t$ are symmetric

C) AA^t and $A + A^t$ are symmetric

D) AA^t, $A + A^t$ and $A - A^t$ are symmetric

E) AA^t, $A + A^t$ and $A - A^t$ are not symmetric

$\boxed{Q6}$ Which of the following sets is a basis for $\mathcal{P}_3$

A) $\{1 + x + x^2, 1 + 2x + 2x^2, -2 - 3x - 3x^2\}$

B) $\{1 + x + x^2, x + x^2, 2\}$

C) $\{x + x^2, x + 1, -x^2 + 1\}$

D) $\{1 + x + x^2, x + x^2, x^2\}$

E) $\{1, 1 + x + x^2\}$

$\boxed{Q7}$ If the point $(1, a, b) \in span\{(1, 1, 0), (2, 1, 1), (2, 3, -1)\}$. Then

A) $a = 0$ and $b = 2$

B) $a = 1$ and $b = 1$

C) $a = 1$ and $b = -1$

D) $a = 2$ and $b = 1$

E) $a = 2$ and $b = -1$

$\boxed{Q8}$ Let $T : \mathbb{R}^4 \to \mathbb{R}^3$,

$$T(a,b,c,d) = \begin{bmatrix} -3 & 1 & 3 & -2 \\ 1 & 1 & -3 & 4 \\ 1 & 3 & -5 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

.

A) $\dim Range(T) = 2$

B) $\dim Ker(T) = 0$

C) $(-3, 3, 5)$ is not in $Range(T)$

D) $Range(T) = \mathbb{R}^3$

E) $Ker(T) = span\{(0, 1, 1, 0)\}$

$\boxed{Q9}$ Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation such that $Kert(T) = \{(0, 0, ..., 0)\}$ and $Range(T) = R^m$. Let M be the standard matrix representation of T. then

A) $n < m$ and $dim(Row(M)) = n$

B) $n > m$ and $dim(Col(M)) = m$

C) It is possible that $det(M) = 0$.

D) $n = m$

E) It is impossible that $M = M^T$

$\boxed{Q10}$ Let $T : R^2 \to P_2$ be a linear transformation such that $T(1,1) = x$ and $T(-1,1) = 2$. Then $T(0,4) =$

A) $2x + 4$

B) $x + 4$

C) $2x + 2$

D) 4

E) $4x + 8$

$\boxed{Q11}$ The following system of linear equations:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$$

A) has a unique solution

B) has infinitely many solutions

C) has $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as a solution

D) has no solution

E) has $\begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix}$ as a solution

Q12 Let $T : R^2 \to \mathbb{P}_2$ be a linear transformation such that $T(a,b) = (a+3b)x + (2a+6b)$. Then the fake standard matrix representation of T is

A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

Q13 Let T as above then:

A) $Ker(T) = \{(0,0)\}, Range(T) = span\{x\}$

B) $Ker(T) = \{(0,0)\}, Range(T) = Span\{x+2\}$

C) $Ker(T) = span\{(1,-3)\}, Range(T) = Span\{x+2\}$

D) $Ker(T) = span\{(-3,1)\}, Range(T) = Span\{x\}$

E) $Ker(T) = span\{(-6,2)\}, Range(T) = Span\{x+2\}$

Q14 Let $M = \begin{bmatrix} a^2 & a^3 \\ 1 & a^4 \end{bmatrix}$. Which of the following statements is **always true**

A) When M is invertible, $M^{-1} = \begin{bmatrix} \dfrac{a}{a^3-1} & \dfrac{-1}{a^3-1} \\ \dfrac{-1}{a^3(a^3-1)} & \dfrac{1}{a(a^3-1)} \end{bmatrix}$

B) $\det M = 0$ only if $a = 1$

C) M is invertible only if $a \neq 0$

D) When M is invertible, $M^{-1} = \begin{bmatrix} \dfrac{1}{a(a^3-1)} & \dfrac{-1}{a^3-1} \\ \dfrac{-1}{a^3(a^3-1)} & \dfrac{a}{a^3-1} \end{bmatrix}$

E) M is row equivalent to I_2

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